# The high pressure $P V T$ properties of deuterium oxide 

Rana A. Fine and Frank J. Millero<br>Rosenstiel School of Marine and Atmospheric Science, University of Miami, Miami, Florida 33149 (Received 30 December 1974)


#### Abstract

The high pressure isothermal compressibilities of deuterium oxide from 5 to $100^{\circ} \mathrm{C}$ and 0 to 1000 bars applied or gauge pressure were determined from sound speed data. These compressibilities were used to derive an equation of state of the form $V^{0} P /\left(V^{0}-V^{P}\right)=B+A_{1} P+A_{2} P^{2}$, where $V^{0}$ and $V^{P}$ are the specific volumes at an applied pressure of zero and $P$; and $B, A_{1}$, and $A_{2}$ are polynomial functions of temperature. The compressibilities derived from this equation of state are consistent with those derived from the sound speed data to $\pm 0.016 \times 10^{-6} \mathrm{bar}^{-1}$ over the entire pressure and temperature range (this is equivalent to $\sim 0.2 \mathrm{~m} \mathrm{sec}^{-1}$ in sound speed). The 1 atm sound-derived compressibilities agree on the average to $\pm 0.06 \times 10^{-6} \mathrm{bar}^{-1}$ with the direct measurements of Millero and Lepple. The $P-V-T$ data from the sound-derived equation are compared with the high pressure work of Bridgman, Kesselman, Juza et al., and Emmet and Millero. Good agreement (average deviation of $\pm 28 \times 10^{-6} \mathrm{~cm}^{3} \mathrm{~g}^{-1}$ ) was found with the recent specific volume measurements of Emmet and Millero. The $\boldsymbol{P}-\boldsymbol{V}-\boldsymbol{T}$ properties of $\mathrm{D}_{2} \mathrm{O}$ are compared to pure water. $\mathrm{D}_{2} \mathrm{O}$ and $\mathrm{H}_{2} \mathrm{O}$ are shown to follow similar trends. Contrary to previous reports, the $\mathrm{D}_{2} \mathrm{O} / \mathrm{H}_{2} \mathrm{O}$ ratios of the specific volumes and specific heats are shown to be functions of both temperature and pressure.


## I. INTRODUCTION

Recently, using the computer technique developed by Wang and Millero, ${ }^{1}$ we derived equations of state for pure water ${ }^{2}$ and seawater. ${ }^{1,3}$ The same technique was used to derive an equation of state and the resulting $P-V-T$ properties for a solution of $99.82 \%$ deuterium oxide ( $\mathrm{D}_{2} \mathrm{O}$ ). Since Mathieson and Conway ${ }^{4}$ demonstrated that the change in sound speed with atomic fraction of $\mathrm{D}_{2} \mathrm{O}$ is nearly linear in mole fraction, the results presented in this paper can easily be applied to any $\mathrm{D}_{2} \mathrm{O}-\mathrm{H}_{2} \mathrm{O}$ mixture. Several investigators examined the $V-T$ properties of $\mathrm{D}_{2} \mathrm{O}$ at $1 \mathrm{~atm} .{ }^{5-16}$ However, at pressures greater than 1 atm , few ${ }^{17-20}$ examined either the $P-V-T$ properties or the similarity of these properties to those of pure water.

There is both a practical and a theoretical significance for studying the similarities of the $P-V-T$ properties of water ( $\mathrm{H}_{2} \mathrm{O}$ is used to denote normal water) and $\mathrm{D}_{2} \mathrm{O}$. On the practical side, the properties of $\mathrm{D}_{2} \mathrm{O}$ can be estimated from $\mathrm{H}_{2} \mathrm{O}$ on the basis of similarity. Comparison of the $P-V-T$ properties of $\mathrm{D}_{2} \mathrm{O}$ and $\mathrm{H}_{2} \mathrm{O}$ is a method that is of use when studying the structural properties of water. ${ }^{21-24}$ The most striking difference between $\mathrm{D}_{2} \mathrm{O}$ and $\mathrm{H}_{2} \mathrm{O}$ is the relationship to life processes. While $\mathrm{H}_{2} \mathrm{O}$ is necessary for life, $\mathrm{D}_{2} \mathrm{O}$ is poisonous to all but the lowest forms. ${ }^{25}$ On the theoretical side, the results of most studies ${ }^{21-24}$ indicated that at the same temperature, $\mathrm{D}_{2} \mathrm{O}$ solutions are more structured than $\mathrm{H}_{2} \mathrm{O}$ solutions. However, Nemathy and Scheraga ${ }^{23}$ showed that the breakdown of this structural order, with an increase in temperature, was more rapid for $\mathrm{D}_{2} \mathrm{O}$ than $\mathrm{H}_{2} \mathrm{O}$.

Since the excellent consistency of the high pressure sound data of Wilson ${ }^{26,27}$ was proven, ${ }^{2}$ we consider our sound-derived equation of state for $\mathrm{D}_{2} \mathrm{O}$ to be more reliable than any of the previous $\mathrm{D}_{2} \mathrm{O}$ work. This work generally applies to both $100 \% \mathrm{D}_{2} \mathrm{O}$ and a $\mathrm{D}_{2} \mathrm{O}-\mathrm{H}_{2} \mathrm{O}$ solution. Where specific numbers are used, an effort is made to identify the particular solution. In Sec. II we review the derivation of our equation, in Sec. III we
compare our results to previous $\mathrm{D}_{2} \mathrm{O}$ work, and in Sec. IV we examine our results relative to $\mathrm{H}_{2} \mathrm{O}$.

## II. EQUATION OF STATE

An equation of state for $\mathrm{D}_{2} \mathrm{O}$ was derived directly from the velocity of sound $c$. The following thermodynamic relationship was used:

$$
\begin{equation*}
\beta=\frac{-1}{V^{P}}\left(\frac{\partial V^{P}}{\partial P}\right)_{T}=\frac{V^{P}}{c^{2}}+\frac{T \alpha^{2} V^{P}}{C_{P}}, \tag{1}
\end{equation*}
$$

in which $\beta$ is the isothermal compressibility $\left[\beta=-1 / V^{P}\right.$ $\left.\times\left(\partial V^{P} / \partial P\right)_{T}\right], V^{P}$ is the specific volume at pressure $P$, $T$ is the absolute temperature, $C_{P}$ is the heat capacity at constant pressure, and $\alpha$ is the expansibility $\left[\alpha=1 / V^{P}\right.$ $\left.\times\left(\partial V^{P} / \partial T\right)_{P}\right]$.

Equation (1) was evaluated using an iterative computer technique developed by Wang and Millero. ${ }^{1}$ To effect this technique for $\mathrm{D}_{2} \mathrm{O}$, the following data were used: the 1 atm densities of $\mathrm{Kell}^{28}$ (estimated error of this equation is $\pm 3 \times 10^{-6} \mathrm{~g} \mathrm{~cm}^{-3}$, estimated accuracy is $\pm 10 \times 10^{-6} \mathrm{~g} \mathrm{~cm}^{-3}$ ); the 1 atm heat capacities of Eucken and Eigen ${ }^{12}$; and the velocity of sound data of Wilson. ${ }^{29}$

Eucken and Eigen ${ }^{12}$ measured the heat capacity at 1 atm of $100 \% \mathrm{D}_{2} \mathrm{O}$ from 20 to $130^{\circ} \mathrm{C}$. The error involved in using $100 \% \mathrm{D}_{2} \mathrm{O}$ heat capacity data in Eq. (1) is at least two orders of magnitude less than the claimed accuracy of Eucken and Eigen. Their claimed accuracy was $0.06 \mathrm{Jg}^{-1} \cdot \mathrm{deg}^{-1}$, which introduces an error of less than $0.005 \times 10^{-6} \mathrm{bar}^{-1}$ in compressibility.

The heat capacity data ${ }^{12}$ below $60^{\circ} \mathrm{C}$ were fitted to an equation and extrapolated down to get values for the range $5-20^{\circ} \mathrm{C}$. Since the heat capacity is such a minor contributor to the compressibility, extrapolation of the data to the lower temperatures where there are no data points of Eucken and Eigen is justified. This extrapolated data was then fitted, along with the data of Eucken and Eigen from 20 to $100^{\circ} \mathrm{C}$, to the following equation:

TABLE I. A comparison of the velocity of sound data ( $\mathrm{m} \mathrm{sec}^{-1}$ ) of Wilson ${ }^{29}$ and Millero et al. ${ }^{30}$ at 1 atm for $99.82 \% \mathrm{D}_{2} \mathrm{O}$.

|  |  | $c$ (Millero | $\Delta c$ (Wilson- <br> $t\left({ }^{\circ} \mathrm{C}\right)$ |
| ---: | :--- | :--- | :--- |
| 4 | $c$ (Wilson) | et al.) | Millero et al.) |
| 10 | 1320.90 | 1320.67 | 0.23 |
| 15 | 1347.49 | 1347.32 | 0.17 |
| 20 | 1384.17 | 1366.75 | 0.21 |
| 25 | 1399.24 | 1383.87 | 0.30 |
| 30 | 1412.33 | 1398.86 | 0.38 |
| 35 | 1423.57 | 1411.88 | 0.45 |
| 40 | 1433.09 | 1432.63 | 0.48 |
| 45 | 1441.01 | 1440.63 | 0.46 |
| 50 | 1447.45 | 1447.20 | 0.38 |
| 55 | 1452.52 | 1452.46 | 0.25 |
| 60 | 1456.32 | 1456.49 | 0.06 |
| 65 | 1458.97 | 1459.38 | -0.17 |
| 70 | 1460.54 | 1461.18 | -0.41 |
| 75 | 1461.14 | 1461.97 | -0.64 |
| 80 | 1460.83 | 1461.78 | -0.83 |
| 85 | 1459.70 | 1460.64 | -0.95 |
| 90 | 1457.82 | 1458.57 | -0.94 |

$$
\begin{align*}
C_{P}^{\circ}= & 4.2765-3.87183 \times 10^{-3} t+6.6500 \times 10^{-5} t^{2} \\
& -7.1819 \times 10^{-7} t^{3}+2.973 \times 10^{-9} t^{4} . \tag{2}
\end{align*}
$$

The maximum deviation of the data of Eucken and Eigen $\left(20-100^{\circ} \mathrm{C}\right.$ ) from Eq. (2) is $0.0015 \mathrm{Jg} \mathrm{g}^{-1} \cdot \mathrm{deg}^{-1}$.

Wilson ${ }^{29}$ measured the velocity of sound in $99.82 \%$ $\mathrm{D}_{2} \mathrm{O}$ from 4 to $98^{\circ} \mathrm{C}$, and from 0 to $\sim 1000$ bar. He reported a probable experimental error of $\pm 0.16 \mathrm{~m} \mathrm{sec}^{-1}$. He fitted his data to a polynomial equation with a standard error of $\pm 0.20 \mathrm{~m} \mathrm{sec}^{-1}$. An error of $\pm 0.20 \mathrm{~m} \mathrm{sec}^{-1}$ in sound velocity will give an error of less than 0.016 $\times 10^{-6} \mathrm{bar}^{-1}$ in compressibility. This resultant small error in compressibility is what enables us to derive a precise equation of state from sound velocities. Although a number of workers ${ }^{10,11,15}$ also measured the velocity of sound in $\mathrm{D}_{2} \mathrm{O}$ at 1 atm , their results are in poor agreement with the work of Wilson. This is expected since these workers ${ }^{10,11,15}$ determined the sound speeds to only $\pm 1 \mathrm{~m} \mathrm{sec}^{-1}$. Recently, Millero et al. ${ }^{30}$ determined the velocity of sound in $\mathrm{D}_{2} \mathrm{O}$ relative to $\mathrm{H}_{2} \mathrm{O}$ from 4 to $90^{\circ} \mathrm{C}$ with a precision of $\pm 0.05 \mathrm{~m} \mathrm{sec}^{-1}$. A comparison of Wilson's ${ }^{29}$ sound velocities with those of Millero et al. ${ }^{30}$ is shown in Table I. This comparison shows that Wilson's sound velocities agree on the average to $\pm 0.45 \mathrm{~m} \mathrm{sec}^{-1}$ (which is equivalent to $\sim \pm 0.03 \times 10^{-6}$ $\mathrm{bar}^{-1}$ in compressibility) with the more reliable work of Millero et al. ${ }^{30}$ This good agreement at 1 atm, in addition to the proven consistency ${ }^{2}$ of the high pressure sound data of Wilson, ${ }^{26,27}$ substantiates our use of Wilson's ${ }^{29}$ sound velocities to derive an equation of state for $\mathrm{D}_{2} \mathrm{O}$.

Our equation of state is of the form of a second degree secant bulk modulus ${ }^{1-3}$ :

$$
\begin{equation*}
K=P V^{0} /\left(V^{0}-V^{P}\right)=B+A_{1} P+A_{2} P^{2}, \tag{3}
\end{equation*}
$$

where $K$ is the secant bulk modulus (at 1 atm absolute pressure or $P=0$ bar applied pressure, $K=1 / \beta$ ); $V^{P}$ and $V^{0}$ are, respectively, the specific volumes at
pressure $P$ and $1 \mathrm{~atm}(0 \mathrm{bar}) ; B, A_{1}$, and $A_{2}$ are polynomial functions of temperature ( $t$ in ${ }^{\circ} \mathrm{C}$ ). The specific volume at $1 \mathrm{~atm}(P=0), V^{0}\left(\mathrm{~cm}^{3} \mathrm{~g}^{-1}\right)$, is taken from Kell ${ }^{28}$ :

$$
\begin{align*}
V^{0}= & \left(1+17.96190 \times 10^{-3} t\right) /\left(1.104690+20.09315 \times 10^{-3} t\right. \\
& -9.24227 \times 10^{-6} t^{2}-55.9509 \times 10^{-9} t^{3}+79.9512 \\
& \left.\times 10^{-12} t^{4}\right) \tag{3a}
\end{align*}
$$

The coefficient $B$ (in bar), which is the reciprocal of the 1 atm compressibility, is given by

$$
\begin{align*}
B= & 1.8607370 \times 10^{4}+1.7026 \times 10^{2} t-2.40556 t^{2} \\
& +1.02703 \times 10^{-2} t^{3}-1.5680 \times 10^{-5} t^{4} . \tag{3b}
\end{align*}
$$

The pressure coefficients $A_{1}$ and $A_{2}$ are given by
$A_{1}=3.129069-4.53919 \times 10^{-3} t+4.3252 \times 10^{-4} t^{2}$

$$
\begin{equation*}
-4.7659 \times 10^{-6} t^{3}+1.6244 \times 10^{-8} t^{4} \tag{3c}
\end{equation*}
$$

$A_{2}=1.07903 \times 10^{-4}-5.5471 \times 10^{-7} t-1.6758 \times 10^{-7} t^{2}$

$$
\begin{equation*}
+2.384 \times 10^{-9} t^{3}-9.301 \times 10^{-12} t^{4} \tag{3d}
\end{equation*}
$$

Figure 1 is a pictorial representation of the residuals or deviations (in bar ${ }^{-1}$ ) of the pressure derivative of the fitted equation (3) from the original sound data. We note that the residuals are apparently random: At zero bar, they are clustered around zero. From 100 to 400 and, not as pronounced, from 600 to 700 bar, they are more heavily weighted on the negative side, whereas at 500 and 800 bar they appear to be more evenly distributed around zero.

The choice of a second degree secant bulk modulus was discussed by Wang and Millero, ${ }^{1}$ Fine and Millero, ${ }^{2}$ and Fine, Wang, and Millero. ${ }^{3}$ When deriving the $P_{-}$ $V-T$ properties, Eq. (3) is convenient to use. Equation (3) can be rearranged to give the specific volume (in $\mathrm{cm}^{3} \mathrm{~g}^{-1}$ ):


FIG. 1. The residuals between compressibilities from Eq. (2) and the sound derived data for each pressure.

TABLE II. The $P-V-T$ properties of $\mathrm{D}_{2} \mathrm{O}$ (99.82\%).

| $P$ (bar) | $\beta\left(\mathrm{bar}^{-1}\right) \times 10^{6}$ |  |  |
| :---: | :---: | :---: | :---: |
|  | $V\left(\mathrm{~cm}^{3} \mathrm{~g}^{-1}\right)$ |  | $\alpha\left(\mathrm{deg}^{-1}\right) \times 10^{6}$ |
| $t=5{ }^{\circ} \mathrm{C}$ |  |  |  |
| 0 | 0.904468 | 51.547 | -114.14 |
| 100 | 0.899880 | 50.177 | -76.14 |
| 200 | 0.895436 | 48.846 | -40.21 |
| 300 | 0.891130 | 47.553 | -6.27 |
| 400 | 0.886959 | 46.297 | 25.75 |
| 500 | 0.882916 | 45.077 | 55.91 |
| 600 | 0.878997 | 43.891 | 84.29 |
| 700 | 0.875199 | 42.738 | 110.93 |
| 800 | 0.871515 | 41.618 | 135.91 |
| 900 | 0.867943 | 40.529 | 159.28 |
| 1000 | 0.864478 | 39.470 | 181.10 |
| $t=25^{\circ} \mathrm{C}$ |  |  |  |
| 0 | 0.905429 | 46.480 | 191.74 |
| 100 | 0.901282 | 45.326 | 206.31 |
| 200 | 0.897257 | 44.213 | 220.47 |
| 300 | 0.893347 | 43.137 | 234.19 |
| 400 | 0.889548 | 42.099 | 247.42 |
| 500 | 0.885856 | 41.095 | 260.13 |
| 600 | 0.882266 | 40.124 | 272.28 |
| 700 | 0.878774 | 39.185 | 283.85 |
| 800 | 0.875378 | 38.277 | 294.81 |
| 900 | 0.872072 | 37.397 | 305.14 |
| 1000 | 0.868854 | 36.546 | 314.83 |
| $t=50^{\circ} \mathrm{C}$ |  |  |  |
| 0 | 0.912656 | 44.859 | 428.72 |
| 100 | 0.908625 | 43.685 | 428.71 |
| 200 | 0.904715 | 42.569 | 429.08 |
| 300 | 0.900921 | 41.506 | 429.75 |
| 400 | 0.897235 | 40.494 | 430.66 |
| 500 | 0.893652 | 39.529 | 431.74 |
| 600 | 0.890168 | 38.608 | 432.95 |
| 700 | 0.886777 | 37.730 | 434.22 |
| 800 | 0.883475 | 36.890 | 435.52 |
| 900 | 0.880257 | 36.089 | 436.80 |
| 1000 | 0.877120 | 35.322 | 438.02 |
| $t=75{ }^{\circ} \mathrm{C}$ |  |  |  |
| 0 | 0.924576 | 46.121 | 602.54 |
| 100 | 0.920384 | 44.789 | 593.65 |
| 200 | 0.916329 | 43.534 | 585.55 |
| 300 | 0.912403 | 42.351 | 578.16 |
| 400 | 0.908598 | 41.234 | 571.40 |
| 500 | 0.904907 | 40.179 | 565.22 |
| 600 | 0.901324 | 39.183 | 559.56 |
| 700 | 0.897842 | 38.240 | 554.37 |
| 800 | 0.894456 | 37.347 | 549.60 |
| 900 | 0.891159 | 36.503 | 545.21 |
| 1000 | 0.887948 | 35.702 | 541.16 |

$$
\begin{equation*}
V^{P}=V^{0}-V^{0} P /\left(B+A_{1} P+A_{2} P^{2}\right) . \tag{4}
\end{equation*}
$$

Differentiation of Eq. (4) with respect to pressure gives the compressibility (in $\mathrm{bar}^{-1}$ ):

$$
\begin{equation*}
\beta=\frac{-1}{V^{P}}\left(\frac{\partial V^{P}}{\partial P}\right)_{T}=\frac{V^{0}\left(B-A_{2} P^{2}\right)}{V^{P}\left(B+A_{1} P+A_{2} P^{2}\right)^{2}} . \tag{5}
\end{equation*}
$$

Differentiation of Eq. (4) with respect to temperature gives the expansibility (in $\mathrm{deg}^{-1}$ ):

$$
\begin{align*}
\alpha= & \frac{1}{V^{P}}\left(\frac{\partial V^{P}}{\partial T}\right)_{P}=\frac{1}{V^{P}}\left(\frac{\partial V^{0}}{\partial T}\right)-\frac{P\left(\partial V^{0} / \partial T\right)}{V^{P}\left(B+A_{1} P+A_{2} P^{2}\right)} \\
& +P V^{0} \frac{(\partial B / \partial T)+P\left(\partial A_{1} / \partial T\right)+P^{2}\left(\partial A_{2} / \partial T\right)}{V^{P}\left(B+A_{1} P+A_{2} P^{2}\right)^{2}} \tag{6}
\end{align*}
$$

For selected temperatures and pressures, we have tabulated in Table II the specific volumes, compressibilities, and expansibilities derived from our equation of state. These values are precise to within $\pm 15$ $\times 10^{-6} \mathrm{~cm}^{3} \mathrm{~g}^{-1}$ in $V^{P}, \pm 2 \times 10^{-6} \mathrm{deg}^{-1}$ in $\alpha$, and $\pm 0.016$ $\times 10^{-6} \mathrm{bar}^{-1}$ in $\beta$. In Sec. III, these $P-V-T$ data are compared to other published results.

## III. COMPARISON OF RESULTS

Since few studies were made on the high pressure $P-V-T$ properties of $\mathrm{D}_{2} \mathrm{O}$, it is not possible to make a number of comparisons to our sound-derived data with direct measurements. Many workers (c.f. Refs. 5, 7, $8,10,13,14,16,18)$ measured or derived equations for the specific volume (or density) of $\mathrm{D}_{2} \mathrm{O}$ at 1 atm . We prefer the equation of $\mathrm{Kell}^{28}$ owing to the reported estimated error of $\pm 3 \times 10^{-6} \mathrm{~g} \mathrm{~cm}^{-3}$ and established accuracy of $\pm 10 \times 10^{-6} \mathrm{~g} \mathrm{~cm}^{-3}$. Also, many of the above mentioned studies do not cover as comprehensive a temperature range.

The most reliable 1 atm compressibilities for $\mathrm{D}_{2} \mathrm{O}$ are from the work of Millero and Lepple. ${ }^{6}$ Using a piezometric technique, they measured the compressibility of $\mathrm{D}_{2} \mathrm{O}$ from 5 to $65^{\circ} \mathrm{C}$ near 1 atm to within $\pm 0.1$ $\times 10^{-6} \mathrm{bar}^{-1}$. Comparisons of the 1 atm compressibilities determined from our equation of state and the work of Millero and Lepple ${ }^{6}$ are shown in Table III. The comparisons show that except at $65^{\circ} \mathrm{C}$ our results are in excellent agreement (average deviation $\pm 0.05_{5} \times 10^{-6}$ $\mathrm{bar}^{-1}$ ) with those of Millero and Lepple.

A number of workers ${ }^{17-20}$ examined the high pressure $P-V-T$ properties of $\mathrm{D}_{2} \mathrm{O}$. The most precise study at low temperatures is the work of Emmet and Millero. ${ }^{17}$ They measured the specific volumes of $(99.8 \%) \mathrm{D}_{2} \mathrm{O}$ (precise to within $\pm 10 \times 10^{-6} \mathrm{~cm}^{3} \mathrm{~g}^{-1}$ ) from 2 to $40^{\circ} \mathrm{C}$ with a high pressure magnetic float densimeter. In the range of our equation of state $\left(5-100^{\circ} \mathrm{C}\right.$ and $\left.0-1000 \mathrm{bar}\right)$, the measurements of Juza et al., ${ }^{18}$ Kesselman, ${ }^{19}$ and Bridgman ${ }^{20}$ provide few data points for comparison. In Fig. 2 we compare the differences in the specific volumes of $\mathrm{D}_{2} \mathrm{O}$ obtained from our equation of state and

TABLE III. A comparison of the compressibility data ( $\mathrm{bar}^{-1}$ ) from the equation of state and Millero and Lepple ${ }^{6}$ at 1 atm.

|  |  | $\beta \times 10^{6}$ <br> (Millero | $\Delta \beta \times 10^{6}$ <br> [Eq. (5)-Millero <br> and Lepple) |
| :--- | :--- | :--- | :--- |
| $t\left({ }^{\circ} \mathrm{C}\right)$ | and Lepple] |  |  |
| 5 | $51.54_{7}$ | 51.49 | 0.06 |
| 10 | $49.80_{2}$ | 49.74 | 0.06 |
| 15 | $48.41_{7}$ | 48.38 | 0.04 |
| 20 | $47.32_{6}$ | 47.37 | -0.04 |
| 25 | 46.48 | 46.52 | -0.04 |
| 30 | $45.84_{1}$ | 45.88 | -0.04 |
| 35 | $45.37_{9}$ | 45.37 | 0.01 |
| 40 | $45.07_{3}$ | 45.10 | -0.03 |
| 45 | $44.90_{5}$ | 44.97 | -0.06 |
| 50 | $44.85_{9}$ | 44.91 | -0.05 |
| 55 | $44.92_{4}$ | 44.98 | -0.06 |
| 60 | 45.09 | 45.16 | -0.07 |
| 65 | $45.34_{9}$ | 45.51 | -0.16 |

V (Ours) -V (Emmet and Millero)


FIG. 2. A comparison of the differences in the specific volumes between the sound derived equation of state and the data of Emmet and Millero. ${ }^{17}$ Unit of contour: $10 \times 10^{-6} \mathrm{~cm}^{3} \mathrm{~g}^{-1}$.
the work of Emmet and Millero. The average deviation of the specific volume differences is $28 \times 10^{-6} \mathrm{~cm}^{3} \mathrm{~g}^{-1}$, and over $90 \%$ of the range ( $0-40^{\circ} \mathrm{C}, 0-1000 \mathrm{bar}$ ) the deviations are within $40 \times 10^{-6} \mathrm{~cm}^{3} \mathrm{~g}^{-1}$. The maximum deviation is $75 \times 10^{-6} \mathrm{~cm}^{3} \mathrm{~g}^{-1}$ at $40^{\circ} \mathrm{C}$ and 1000 bar. The larger deviations at the high temperatures and pressures are probably a result of errors in the direct measurements due to the nonequilibrium of the magnetic float. ${ }^{31}$ The overall agreement is good and provides support for the validity of our equation of state.

Using a sylphon method, Bridgman ${ }^{20}$ measured the specific volume of $99.9 \% \mathrm{D}_{2} \mathrm{O}$ from -20 to $100^{\circ} \mathrm{C}$ and 0 to $\sim 12000$ bar. A comparison of our results (adjusted to $99.9 \% \mathrm{D}_{2} \mathrm{O}$ ) with those of Bridgman are shown in Table IV. The agreement is not very good; however, over most of the range the differences are within the experimental error of Bridgman's direct measurements $\left(\sim 500 \times 10^{-6} \mathrm{~cm}^{3} \mathrm{~g}^{-1}\right)$. Kesselman ${ }^{19}$ derived an equation of state for $\mathrm{D}_{2} \mathrm{O}$ for the range $20-380^{\circ} \mathrm{C}$ and $0-500 \mathrm{bar}$. He states that the average deviations of his equation do not exceed the experimental error of Bridgman ${ }^{20}$ and Kirillin and Ulybin. ${ }^{32}$ As a consequence, the agreement between Kesselman's and our equation of state is very poor ( $\sim 1000 \times 10^{-6} \mathrm{~cm}^{3} \mathrm{~g}^{-1}$ in specific volume). Poor agreement ( $1500 \times 10^{-6} \mathrm{~cm}^{3} \mathrm{~g}^{-1}$ ) was also found with the data of Juza et al. ${ }^{18}$ They measured the specific volume of $\mathrm{D}_{2} \mathrm{O}$ at 80 and $100^{\circ} \mathrm{C}, 499$ and 999 bar.

## IV. COMPARISONS OF THE P-V-T PROPERTIES OF $\mathrm{D}_{2} \mathrm{O}$ AND $\mathrm{H}_{2} \mathrm{O}$

Since the viscosity, melting point, temperature of maximum density, and heat capacity are all higher in $\mathrm{D}_{2} \mathrm{O}$ than in $\mathrm{H}_{2} \mathrm{O}$, Nemethy and Scheraga ${ }^{33}$ (and others) proposed that there is more structural order in $\mathrm{D}_{2} \mathrm{O}$ than in $\mathrm{H}_{2} \mathrm{O}$, or that the degree of hydrogen bonding is higher in $\mathrm{D}_{2} \mathrm{O}$ than in $\mathrm{H}_{2} \mathrm{O}$. In recent years a number of workers ${ }^{21-23}$ have examined the structural properties of $\mathrm{D}_{2} \mathrm{O}$ relative to $\mathrm{H}_{2} \mathrm{O}$. Although the high pressure $P-V-T$ properties of $\mathrm{D}_{2} \mathrm{O}$ and $\mathrm{H}_{2} \mathrm{O}$ are similar, the small differences that do occur can be very useful in

TABLE IV. A comparison of the specific volume data from the equation of state and Bridgman. ${ }^{20}$

|  | $\Delta V \times 10^{6} \mathrm{~cm}^{3} \mathrm{~g}^{-1}$ |  |  |
| ---: | :---: | :---: | :---: |
| $t\left({ }^{\circ} \mathrm{C}\right)$ | 0 bar | 499 bar | 999 bar |
| 20 | -455 | -1045 | -1569 |
| 40 | 226 | -562 | -1027 |
| 50 | 262 | 270 | -1037 |
| 60 | -113 | -350 | -990 |
| 80 | 62 | -40 | -993 |
| 100 | $\cdots$ | 272 | -176 |

the examination of the structure of $\mathrm{H}_{2} \mathrm{O}$. It is not the purpose of this paper to discuss these differences in great detail; however, it is useful to examine some of them. Above the temperature of maximum density the $P-V-T$ properties of $\mathrm{D}_{2} \mathrm{O}$ and $\mathrm{H}_{2} \mathrm{O}$ are similar. For example, the specific volumes increase as the temperature is increased, and decrease as the pressure is increased; the expansibilities increase as the temperature is increased and increase as the pressure is increased; the compressibilities decrease as the pressure is increased, and go through a minimum when plotted versus temperature. In this section we examine the effect of temperature and pressure on the $P-V-T$ properties of $\mathrm{D}_{2} \mathrm{O}$ and $\mathrm{H}_{2} \mathrm{O}$. The $P-V-T$ properties for $\mathrm{D}_{2} \mathrm{O}$ are from the equation of state (3), and the properties for $\mathrm{H}_{2} \mathrm{O}$ are from the equation derived by Fine and Millero. ${ }^{2}$

The effect of temperature on the specific volume, compressibility, expansibility, and specific heats at constant volume and pressure of $\mathrm{D}_{2} \mathrm{O}$ and $\mathrm{H}_{2} \mathrm{O}$ are examined graphically. In Fig. 3 the specific volumes of $\mathrm{D}_{2} \mathrm{O}$ and $\mathrm{H}_{2} \mathrm{O}$ are plotted versus temperature at two pressures ( $P=0$ and 1000 bar ). The curves are similar for $\mathrm{D}_{2} \mathrm{O}$ and $\mathrm{H}_{2} \mathrm{O}$ with $\mathrm{H}_{2} \mathrm{O}$ having the larger volumes. The compressibilities of $\mathrm{D}_{2} \mathrm{O}$ and $\mathrm{H}_{2} \mathrm{O}$ (Fig. 4) show the minimum in the middle temperature range at both pressures ( 0 and 1000 bar).

The expansibilities of $\mathrm{D}_{2} \mathrm{O}$ and $\mathrm{H}_{2} \mathrm{O}$ (Fig. 5) plotted versus temperature are also graphically similar. For both liquids the expansibility values at 0 and 1000 bar converge in the middle temperature range. The conver-


FIG. 3. The specific volumes of $\mathrm{D}_{2} \mathrm{O}$ and $\mathrm{H}_{2} \mathrm{O}$ as a function of temperature at 0 and 1000 bar pressure.


FIG. 4. The compressibilities of $\mathrm{D}_{2} \mathrm{O}$ and $\mathrm{H}_{2} \mathrm{O}$ as a function of temperature at 0 and 1000 bar pressure.

## gence is at approximately $50^{\circ} \mathrm{C}$ for $\mathrm{H}_{2} \mathrm{O}$ and $\mathrm{D}_{2} \mathrm{O}$.

The heat capacities at constant volume ( $\mathrm{C}_{V}$ ) plotted versus temperature show fairly similar patterns for $\mathrm{D}_{2} \mathrm{O}$ and $\mathrm{H}_{2} \mathrm{O}$ (Fig. 6). For the $\mathrm{C}_{V}$ of $\mathrm{D}_{2} \mathrm{O}$ at pressures of 0 and 1000 bar, there is a stronger negative slopeespecially at higher temperatures-than for $\mathrm{H}_{2} \mathrm{O}$. The differences between the slopes of the $C_{V}$ for $\mathrm{H}_{2} \mathrm{O}$ and for $\mathrm{D}_{2} \mathrm{O}$ may be due to structural differences between the two liquids. We may be seeing the more rapid breakdown of the structural order in $\mathrm{D}_{2} \mathrm{O}$ than $\mathrm{H}_{2} \mathrm{O}$ as the temperature is increased. This was previously shown by Nemethy and Scheraga. ${ }^{33}$ In Fig. 7 we see similar curves for the $C_{P}$ of $\mathrm{H}_{2} \mathrm{O}$ and $\mathrm{D}_{2} \mathrm{O}$ at 0 and 1000 bar pressure.

Figure 8 is a plot of the temperatures of maximum density of $\mathrm{D}_{2} \mathrm{O}$ and of $\mathrm{H}_{2} \mathrm{O}$ versus pressure. We note the similar decrease in the temperatures of maximum density as the pressure is increased for both liquids. The slope of the fitted $\mathrm{D}_{2} \mathrm{O}$ temperatures is slightly weaker than that of $\mathrm{H}_{2} \mathrm{O}$. This may be another indication of a $\mathrm{D}_{2} \mathrm{O}-\mathrm{H}_{2} \mathrm{O}$ structural difference, and a less rapid breakdown of the $\mathrm{D}_{2} \mathrm{O}$ structural order as the pressure is increased.


FIG. 5. The expansibilities of $\mathrm{D}_{2} \mathrm{O}$ and $\mathrm{H}_{2} \mathrm{O}$ as a function of temperature at 0 and 1000 bar pressure.


FIG. 6. The heat capacities at constant volume of $\mathrm{D}_{2} \mathrm{O}$ and $\mathrm{H}_{2} \mathrm{O}$ as a function of temperature at 0 and 1000 bar pressure.

We also examined the effect of pressure on the $P-V-T$ properties of $\mathrm{D}_{2} \mathrm{O}$ and $\mathrm{H}_{2} \mathrm{O}$. Once again we looked at the specific volume, compressibility, expansibility, and specific heats at constant volume and pressure. Over the range of both equations we found that the $P-V-T$ properties of $\mathrm{D}_{2} \mathrm{O}$ and $\mathrm{H}_{2} \mathrm{O}$ followed the same patterns when plotted versus pressure.

Some workers tried to show that at each temperature the $\mathrm{D}_{2} \mathrm{O} / \mathrm{H}_{2} \mathrm{O}$ ratios of certain $P-V-T$ properties are independent of pressure. Rivkin ${ }^{8}$ showed that from 50 to $100^{\circ} \mathrm{C}$ and from 0 to 100 bar the $\mathrm{D}_{2} \mathrm{O} / \mathrm{H}_{2} \mathrm{O}$ density ratios were independent of pressure to within $510 \times 10^{-6}$. We found that for 50 and $75^{\circ} \mathrm{C}$, from 0 to 1000 bar the ratios were within $674 \times 10^{-6}$ and $481 \times 10^{-6}$, respectively.

Figure 9 examines the $\mathrm{D}_{2} \mathrm{O} / \mathrm{H}_{2} \mathrm{O}$ ratios of the densities versus pressure at 5 and $50^{\circ} \mathrm{C}$. The figure shows the linearity of the ratios versus pressure, with larger slopes at lower temperatures. In view of recent work (this paper, Emmet and Millero, ${ }^{17}$ and Fine and Mil-


FIG. 7. The heat capacities at constant pressure of $\mathrm{D}_{2} \mathrm{O}$ and $\mathrm{H}_{2} \mathrm{O}$ as a function of temperature at 0 and 1000 bar pressure.


FIG. 8. The temperatures of maximum density of $\mathrm{D}_{2} \mathrm{O}$ and $\mathrm{H}_{2} \mathrm{O}$ as a function of pressure.
lero ${ }^{2}$ ), we do not consider that these ratio differences for the densities (or specific volumes) are independent of pressure.

Rivkin and Egorov ${ }^{9}$ also examined the $\mathrm{D}_{2} \mathrm{O} / \mathrm{H}_{2} \mathrm{O}$ ratios. They examined them for heat capacities from 20 to $100^{\circ} \mathrm{C}$ and 0 to 100 bar pressure. They concluded that the ratios of the heat capacities were independent of pressure to within 0.014 . Our results showed similar ratios from 0 to 1000 bar. However, considering the reported accuracy of recent heat capacity measurements, for example on seawater by Millero et al. ${ }^{34}( \pm 0.0005 \mathrm{~J}$ $\mathrm{g}^{-1} \cdot \mathrm{deg}^{-1}$ ), the $\mathrm{D}_{2} \mathrm{O} / \mathrm{H}_{2} \mathrm{O}$ heat capacity ratios are not independent of pressure. We assert that neither the $\mathrm{D}_{2} \mathrm{O}$ / $\mathrm{H}_{2} \mathrm{O}$ ratios of the densities nor the heat capacities are independent of pressure (or temperature).

We conclude that the most precise high pressure $P$ -$V-T$ properties can be derived from our equation of state for $\mathrm{D}_{2} \mathrm{O}$. This equation was derived from the velocity of sound data of Wilson. ${ }^{29}$ There is good agreement of the $P-V-T$ properties from this sound-derived equation with the recent direct experimental work of Emmet and Millero. ${ }^{17}$ We found a similarity in the $\mathrm{D}_{2} \mathrm{O}$ and $\mathrm{H}_{2} \mathrm{O}$ curves plotted versus temperature and pressure for all the $P-V-T$ properties. It was previously reported ${ }^{8,9}$ that the $\mathrm{D}_{2} \mathrm{O} / \mathrm{H}_{2} \mathrm{O}$ specific volume and heat capacity ratios at each temperature were independent of pressure. We found this to be incorrect. Our finding was primarily due to the greater precision of our data. The differences between the $P-V-T$ properties for $\mathrm{D}_{2} \mathrm{O}$ and $\mathrm{H}_{2} \mathrm{O}$, in general, were found to be both temperature and pressure dependent. The high pressure $P-V-T$ properties of $\mathrm{D}_{2} \mathrm{O}$ and $\mathrm{H}_{2} \mathrm{O}$ are similar in that they follow the same trends. However, examination of the properties, with the precision we achieved, allows a clearer understanding of the differences. This examination may also lead to further understanding of the $\mathrm{D}_{2} \mathrm{O}-\mathrm{H}_{2} \mathrm{O}$ structural differences.

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FIG. 9. The $\mathrm{D}_{2} \mathrm{O} / \mathrm{H}_{2} \mathrm{O}$ density ratios as a function of pressure at 5 and $50^{\circ} \mathrm{C}$.

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